**Five New Rules for Predicate Logic Proofs**

You can think of the first four rules as ways of gaining or losing quantifiers.

**UI** allows you to drop a universal quantifier and replace all the variables bound by it to any constant or variable. Think of a variable you instantiate to as being an arbitrarily chosen domain element.

**EG** is also almost unrestricted. You can replace a constant or a variable with a variable and bind it with an existential quantifier. The only restriction is that you can’t generalize to a variable that’s already contained in the formula. Here’s an example of a proof that violates the restriction.

1. ∃x∃y(Fx∙∼Fy) p
2. ∃y(Fx∙∼Fy) EI 1
3. Fx∙∼Fy EI 2
4. ∃y(Fy∙∼Fy) EG3 (INVALID)

There are restrictions on the next two rules, UG and EI.

**EI** allows you to drop an existential quantifier and change the variables bound by it to a new variable—one that doesn’t occur free anywhere earlier in the proof. The variable is not an arbitrarily selected domain element, but rather one chosen to satisfy the condition of the existentially quantified claim. Here’s an example in English:

Some mules have successfully reproduced.

Call one such mule ‘lucky.’

So, Lucky is a mule that successfully reproduced.

There isn’t really one particular mule picked out by ‘Lucky’. It’s not like a name or a constant letter. ‘Lucky’ is a kind of variable. Also, it doesn’t pick out any arbitrary domain element, but rather it picks out any of those that satisfy a certain condition. In this case the condition is: a mule that successfully reproduced.

You can tell that we shouldn’t be able to universally generalize on that kind of variable

**UG** allows you to bind some variable with a universal quantifier *as long as the variable was an arbitrarily chosen domain element*. If the variable was free in an EI line (this includes more than just the variables that came from EI) or appeared in an un-discharged assumption for CP or AP (these assumptions are like EI lines) then it isn’t arbitrarily chosen; it’s more like ‘Lucky’ in the silly example above.

Consult the textbook for (a multitude) of examples of both correct and incorrect applications of these rules.

QN allows you to “push a negation sign through” a quantifier provided you flip the quantifier. In order to use the instantiation rules (UI and EI) you must have a line that begins with a quantifier. You cannot, for example, use UI like this:

1. ∼∀x Px p
2. ∼Pa UI 1 (INVALID)

Just because not everything is physical doesn’t guarantee that Andy is not physical. A negated universal quantifier is equivalent to an existentially quantified negation, and you cannot existentially instantiate (EI) to a constant. Line 1 of the failed proof is equivalent to:

1. ∃x∼Px QN 1

That equivalence is one of the components of QN. For some it is helpful to think of a universally quantified sentence as a kind of conjunction and an existentially quantified sentence as a kind of disjunction.

∀x(Fx⊃Gx) says something like (Fa⊃Ga) ∙ (Fb⊃Gb) ∙ (Fc⊃Gc)…

∃x(Fx∙∼Gx) says something like (Fa∙∼Ga) ∨ (Fb∙∼Gb) ∨ (Fc∙∼Gc)…

With that understanding of the quantifiers, QN should remind you of DeMorgan’s rule. It says that the negation of a conjunction is the disjunction of negations , and the negation of a disjunction is a conjunction of negations. Stop here, think for a second, and make sure you see the relationship between DeMorgan’s and QN.

Here are two sample arguments that illustrate (depending on your symbolization) the use of QN.

From Aquinas’ Second Way:

Causes precede their effects.

Nothing precedes itself.

So, nothing can cause itself.

Hint: Use relational predicates.

Frank Jackson’s Knowledge Argument against Physicalism (the view that everything is physical):

If everything is physical then all knowledge is physical knowledge.

Some knowledge is not physical knowledge.

So, something is not physical.

By the way, to capture Jackson’s argument, you should use a single predicate for ‘is physical knowledge’. What he means by it isn’t captured by using a conjunction formed with the predicates ‘is physical’ and ‘is knowledge’.

Also try #10 and #12 in exercise 9-3 on page 224.