Worksheet - Fluids in motion

**1.** The water flowing through pipe A flows out through pipes B, C, and D. The diameter of a cross section of pipe A is 2cm, and all of other pipes have a diameter of 1.4cm. The volume flow rate in pipes B, C, and D are 28L/min, 18L/min, and 10L/min respectively.



**a)** What is the volume flow rate of pipe A?

$$R\_{A}=\left(28+18+10\right)\frac{L}{min}=56\frac{L}{min}=\frac{9.33x10^{-4}m^{3}}{s}$$

**b)** What is the pipe A to pipe B ratio of the speed of water?

$$\frac{v\_{A}}{v\_{B}}=\frac{R\_{A}/A\_{A}}{R\_{B}/A\_{B}}=\frac{\left(\frac{9.33x10^{-4}m^{3}}{s}\right)/\left(0.25π\left(2x10^{-2}m\right)^{2}\right)}{\left(\frac{4.67x10^{-4}m^{3}}{s}\right)/\left(0.25π\left(1.4x10^{-2}m\right)^{2}\right)}=0.98$$

**2.** A liquid of density 820 kg/m3 flows through a horizontal pipe that has a cross sectional area of 2.8x10-2 m2 in region A and a cross sectional area of 8.1x10-2 m2 in region B. The pressure difference between the two regions is 6.1x103 Pa.

**a)** What is the volume flow rate? Since it's horizontal the potential terms cancel out.

$$R=A\_{1}v\_{1} R=A\_{2}v\_{2} P\_{1}+\frac{1}{2}ρv\_{1}^{2}=P\_{2}+\frac{1}{2}ρv\_{2}^{2}$$

We need to combine the continuity equation with Bernoulli's principal.

We'll do this by solving both volume flow rate equations for velocity and insert them into the Bernoulli equation.

$v\_{1}=\frac{R}{A\_{1}}$ and $v\_{2}=\frac{R}{A\_{2}}$ so $P\_{1}+\frac{1}{2}ρ\left(\frac{R}{A\_{1}}\right)^{2}=P\_{2}+\frac{1}{2}ρ\left(\frac{R}{A\_{2}}\right)^{2}$

Next we solve for our objective, which is the volume flow rate represented as R in the equations.

$$P\_{1}-P\_{2}=\frac{1}{2}ρ\left(\frac{R}{A\_{2}}\right)^{2}-\frac{1}{2}ρ\left(\frac{R}{A\_{1}}\right)^{2}\rightarrow ∆P=\frac{1}{2}ρR^{2}\left(\frac{1}{A\_{2}^{2}}-\frac{1}{A\_{1}^{2}}\right)$$

$$volume flow rate=R=A\_{1}A\_{2}\sqrt{\frac{2∆P}{ρ\left(A\_{2}^{2}-A\_{1}^{2}\right)}}$$

$$=\left(2.8x10^{-2}m^{2}\right)\left(8.1x10^{-2}m^{2}\right)\sqrt{\frac{2\left(6.1x10^{3}Pa\right)}{\left(820\frac{kg}{m^{3}}\right)\left(\left(8.1x10^{-2}m^{2}\right)^{2}-\left(2.8x10^{-2}m^{2}\right)^{2}\right)}}$$

$$volume flow rate=0.115\frac{m^{3}}{s}$$

**b)** What is the mass flow rate?

$$mass flow rate= \left(820\frac{kg}{m^{3}}\right)\left(0.115\frac{m^{3}}{s}\right)=94.3\frac{kg}{s}$$